Asset Financing with Credit Risk

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Asset-Backed Financing

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This talk focuses on financing depreciating equipment, with particular focus on aircraft financing (aircraft mortgages) and their securitization (EETCs)



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- Export credit agencies (credit guarantees to private market): In the event of airline default, the guarantor (ECA) pays off the outstanding principal plus accrued interest and takes over the underlying asset backed loan. Similar to a CDS written on an asset backed loan.

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- Aircraft manufacturers through their credit operations

Modeling Considerations

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Modeling the Risks

- Model for the underlying asset: depreciating equipment with significant and time (age) dependent secondary market volatility
- Model to evaluate secured asset financing (loans, leases, securitization vehicles, credit guarantees) consistently with unsecured financing (senior unsecured corporate debt, Credit Default Swaps (CDS))

What does the underlying asset look like?

- Ascend historical current market value (CMV) data for 14 models:

A320-200 1988-2008	B747-400 1988-2007
A340-300 1992-2008	B757-200 1982-2004
ATR42-300 1985-96	B767-300ER 1986-2007
B727-200A 1972-83	CRJ-100ER 1991-2001
B737-300 1984-99	Dash8-100 1983-2003
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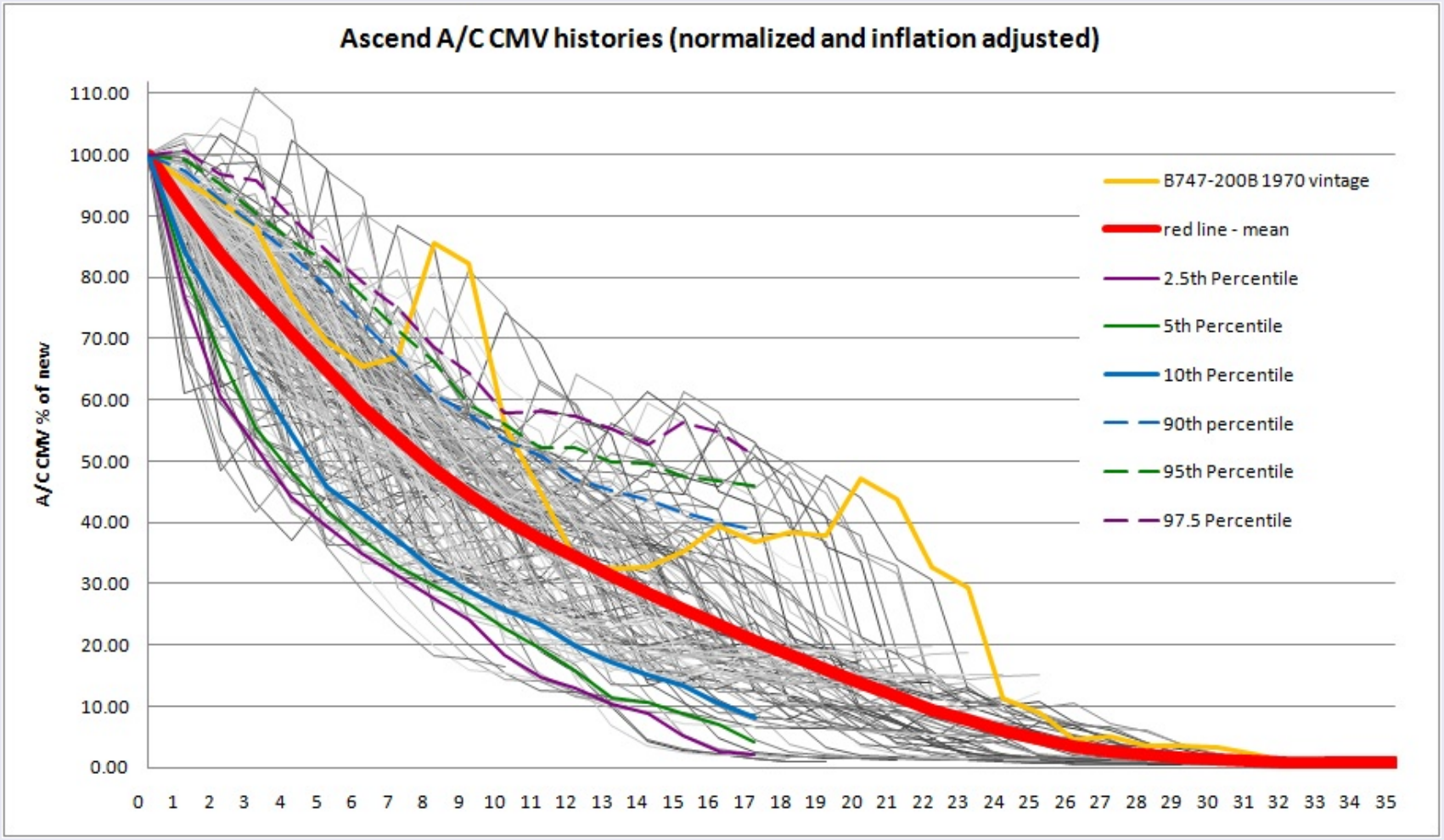
- From 1970 to mid-2008, 230 model/vintage time series of annual CMV appraisals, 3,528 data points
- Data normalized to 100 for initial appraisal and deflated by CPI to adjust for inflation
- For estimation, each time series (i.e. B737-300, vintage 1994 and DC10-30, vintage 1985) was considered to be a sample path of the same random variable (the asset value), and appraisals were binned by age (i.e. all appraisals for 3 year-old models were binned together)

Ascend CMV Histories (Inflation Adjusted)

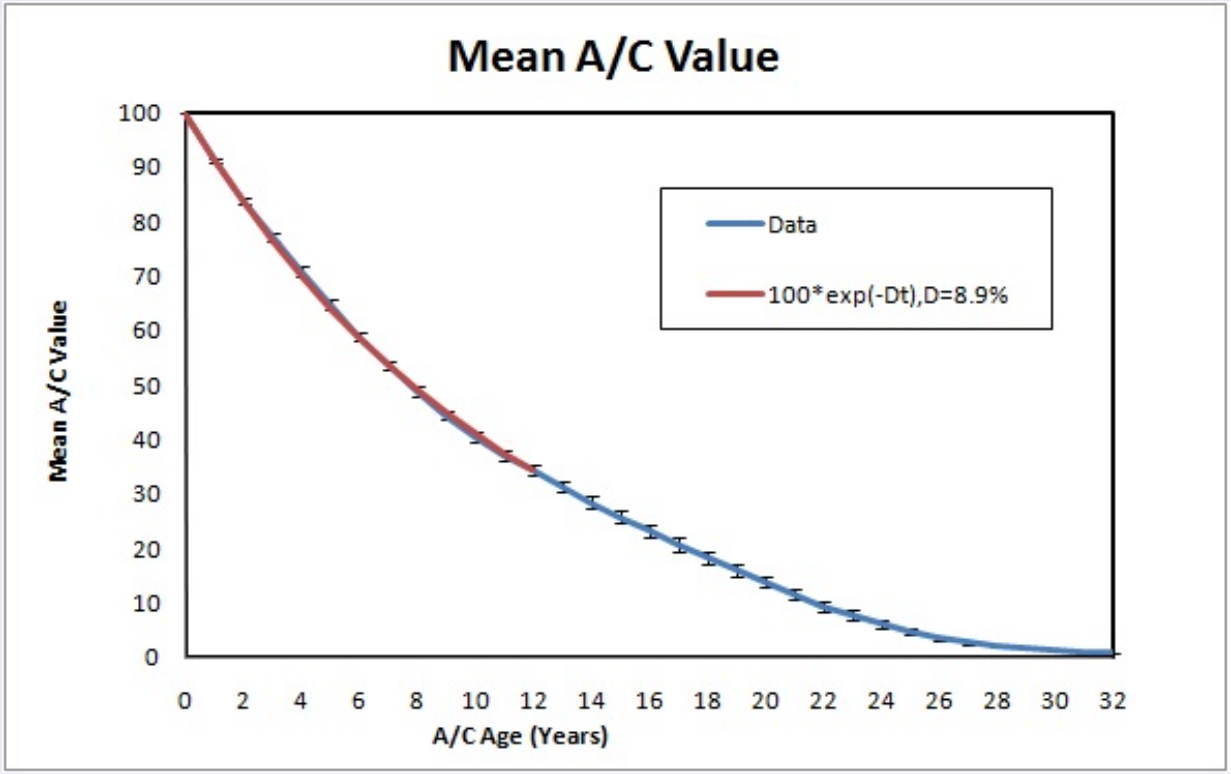


Residual Value Curve (RVC): Mean Value as a Function of Age

- This is the Forward Curve under the $\mathbb{P}$-measure:

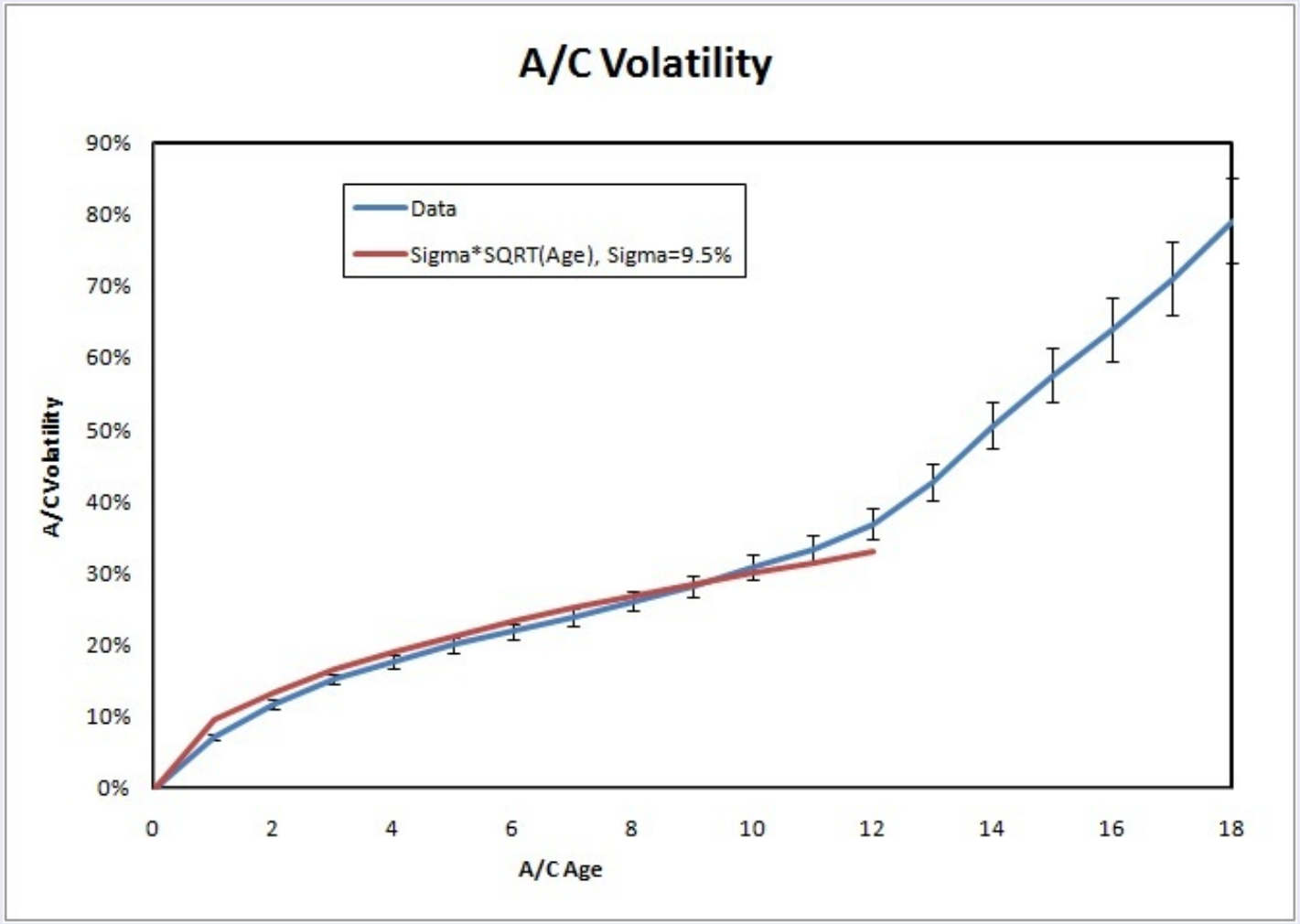
$$F^{\mathbb{P}}(0,t) = \mathbb{E}^{\mathbb{P}}[S_t]$$

- Tracks the expected depreciation of the aircraft with age



Volatility as a Function of Age

We will later identify this with the asset volatility $\Sigma(t)$



Stochastic Model for Depreciating Equipment

- Equipment is new at $t = 0$ with price S_0. Secondary Market used equipment price dynamics under the physical probability measure $\mathbb{P}$:

$$dS_t = -D_t S_t dt + S_t \sigma_S(t) \cdot dW_t^{\mathbb{P}}$$

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- Assuming the absence of arbitrage, under a change of measure $dW_t^{\mathbb{P}} = dW_t^{\mathbb{Q}} - \eta_t dt$, we obtain the risk-neutral ($\mathbb{Q}$) dynamics:

$$\begin{aligned} dS_t &= (r_t - \ell_t) S_t dt + S_t \sigma_S(t) \cdot dW_t^{\mathbb{Q}} \\ \sigma_S(t) \cdot \eta_t &= \ell_t - r_t - D_t \end{aligned}$$

where ℓ_t is the stochastic default-free instantaneous lease rate (short lease rate).

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- If we assume that the equipment has a finite lifetime T, at which point it is sold for its scrap value S_T (which we assume to independent of the model dynamics), we obtain that:

$$S_t = S_T \mathbb{E}_t^{\mathbb{Q}} \left[e^{\int_t^T (r_s - \ell_s) ds} \right]$$

Stochastic Model for Depreciating Equipment

- We assume that the stochastic short lease-rate follows a Vasicek process with time-dependent parameters under $\mathbb{Q}$:

$$d\ell_t = \kappa_\ell(\theta_\ell(t) - \ell_t)dt + \sigma_\ell(t) \cdot dW_t^{\mathbb{Q}},$$

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- Express ℓ_t in terms of the OU process with zero long-run mean X_t^ℓ and deterministic Δ_t^ℓ:

$$\ell_t = \Delta_t^\ell + X_t^\ell, \qquad \Delta_0^\ell = \ell_0, \;\; X_0^\ell = 0$$

$$\begin{aligned} dX_t^\ell &= -\kappa_\ell X_t^\ell dt + \sigma_\ell(t) \cdot dW_t^\ell. \\ X_t^\ell &= X_s^\ell + \int_s^t e^{-\kappa_\ell(t-u)} \sigma_\ell(u) \cdot dW_u^{\mathbb{Q}}, \end{aligned}$$

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- The asset dynamics are therefore lognormal and in terms of X_t^ℓ and the risk-neutral forward curve $F^{\mathbb{Q}}(0,t)$:

$$S_t = F^{\mathbb{Q}}(0,t) e^{\frac{X_t^\ell}{\kappa_\ell}(1-e^{-\kappa_\ell(T-u)}) - \frac{1}{2}\Sigma^2(t)}$$

Stochastic Model for Depreciating Equipment

- This lognormal specification fixes the asset volatility function such that:

$$\begin{aligned}\sigma_S(t) &= \sigma_\ell(t,T) \equiv \frac{\sigma_\ell(t)}{\kappa_\ell}(1-e^{-\kappa_\ell(T-t)}) \\ \Sigma^2(t) &= \frac{1}{\kappa_\ell^2}(1-e^{-\kappa_\ell(T-t)})^2 \int_0^t \sigma_\ell^2(s) e^{-2\kappa_\ell(t-s)} ds\end{aligned}$$

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- We estimate the model from the time series data on used aircraft prices, $F^{\mathbb{P}}(0,t) = \mathbb{E}^{\mathbb{P}}[S_t]$, by exploiting the change-of-measure relationship:

$$F^{\mathbb{Q}}(0,t) = F^{\mathbb{P}}(0,t)e^{-\eta\int_0^t \sigma_\ell(s,T)e^{-\kappa_\ell(t-s)}ds}$$

which along with the estimated squared volatility function, yields that:

$$\boxed{S_t = F^{\mathbb{P}}(0,t)e^{-\eta\int_0^t \sigma_\ell(s,T)e^{-\kappa_\ell(t-s)}ds}e^{\frac{X_t^\ell}{\kappa_\ell}(1-e^{-\kappa_\ell(T-u)})-\frac{1}{2}\Sigma^2(t)}}$$

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- We observe the relationship between the forward curve $F^{\mathbb{Q}}(0,t)$ (expected future value under $\mathbb{Q}$) and the RVC:

$$\boxed{F^{\mathbb{Q}}(0,t) = F^{\mathbb{P}}(0,t)e^{-\eta \int_0^t \sigma_\ell(s,T)e^{-\kappa_\ell(t-s)}ds}}$$

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  We call this Risk Adjusted Residual Value Curve (RA-RVC).
- We will **imply** the market price of risk η from the market prices of **Enhanced Equipment Trust Certificates (EETCs)** - securitized aircraft mortgages which are publicly traded, and therefore provide an opportunity to estimate the MPR.

Valuing Aircraft Loans

- Because of the risk of default, the value of the loan consists of two parts: the coupon stream up until the minimum of the time of default, τ, or maturity, and a recovery amount in the event of default.

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$$\begin{aligned} R(\tau) &= \min\{P(\tau)+I(\tau), S_\tau - TC\} + \pi \max\{P(\tau)+I(\tau)+TC-S_\tau, 0\} \\ &= P(\tau)+I(\tau) - (1-\pi)\max\{P(\tau)+I(\tau)+TC-S_\tau, 0\}, \end{aligned}$$

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- We may therefore write the value of a loan of maturity T^* as,

$$M = \mathbb{E}\left[\sum_{k=0}^{N} e^{-\int_0^{t_k} r_s ds} C_k \mathbf{1}_{\{\tau > t_k\}}\right] + \mathbb{E}\left[e^{-\int_0^{\tau} r_s ds} \mathbf{1}_{\{\tau \le T^*\}} R(\tau)\right]$$

Unsecured Credit Risk Modeling

Default Intensity

- We assume the existence of stochastic default intensity $\lambda(t)$ which has the dynamics of a Vasicek process with time-dependent parameters under $\mathbb{Q}$:

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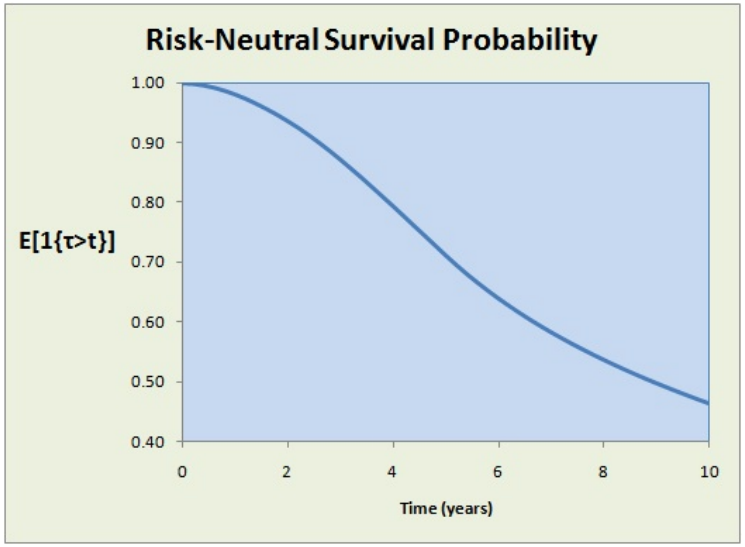

Figure: Implied from CDS quotes for Continental Airlines 4/10/2007

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- With the model architecture in place, all expectations in the loan valuation can be computed analytically through change of measure techniques similar to those in FX options.

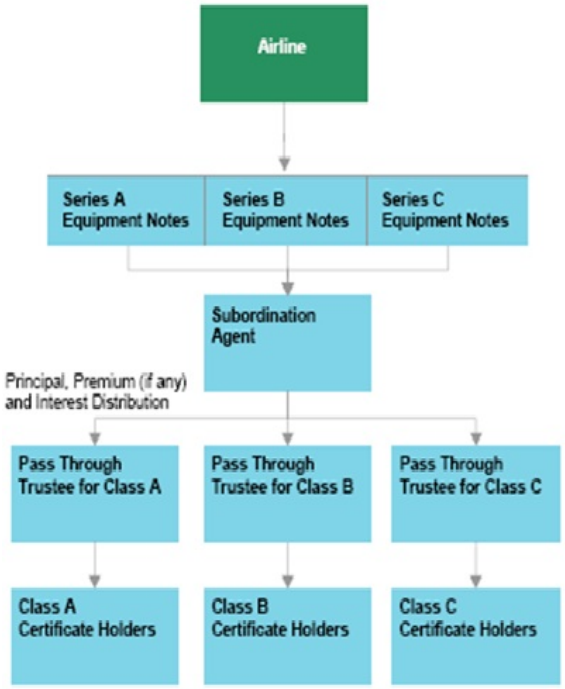
$$\begin{aligned} M &= \sum_{k=1}^{N} \mathbb{E}\left[e^{-\int_0^{t_k} r_s ds} 1_{\{\tau > t_k\}} C_k\right] \\ &+ \sum_{k=1}^{N} \mathbb{E}[1_{\{t_{k-1} < \tau \leq t_k\}} e^{-\int_0^{\tau} r_s ds} (1 + (\tau - t_{k-1})L) P_{k-1}] \\ &- (1-\pi) \sum_{k=1}^{N} \mathbb{E}\left[e^{-\int_0^{\tau} r_t dt} 1_{\{t_{k-1} < \tau \leq t_k\}} (K_k(\tau) - S(\tau))^+\right] \end{aligned}$$

where L is the loan rate and $K_k(\tau) = P_{k-1} + (\tau - t_{k-1}) L\, P_{k-1} + TC$.

Enhanced Equipment Trust Certificates

Enhanced Equipment Trust Certificates (EETCs)

- A form of corporate bond
  - Secured by a pool of aircraft, which are purchased by the airline with the proceeds
  - Rated (often significantly) higher than the underlying airline's unsecured debt



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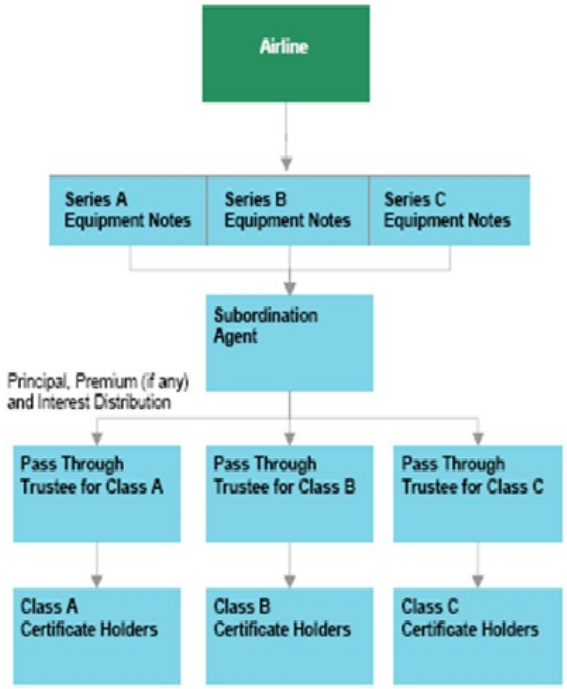
- **Features**
  - Constant levels of over-collateralization
  - Perfected security interest in the aircraft that is enforceable in a timely manner
  - Section 1110 protection: automatic 60 day repossession of aircraft outside of normal bankruptcy proceedings
  - Parsed into multiple tranches, each with their own LTV ratio, loan rate, amortization schedule, maturity, credit rating and seniority (senior, mezzanine, junior/subordinate certificates)



Terms of EETC issued by Continental Airlines in 2007

Summary of Terms of Certificates

	Class A Certificates	Class B Certificates	Class C Certificates
Aggregate Face Amount	$756,762,000	$221,850,000	$168,198,000
Interest Rate	5.983%	6.903%	7.339%
Ratings:			
Moody's	Baa1	Ba2	B1
Standard & Poor's	A	BBB-	B+

- Rating of the Airline:
  - Moodys: B3
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- Low LTV + perfected security interest leads to higher ratings for tranches:

$$\text{Aircraft appraisal: } \$1,146,810,000 \Longrightarrow \text{LTV} = 73.79\%$$

Valuing EETCs within the Framework

- The recovery payment for the senior tranche is determined the same way as for a standard loan. The recovery for the next tranche is determined similarly but now we need to subtract the recovery paid to the senior tranche from the proceeds of the sale of the aircraft. Similarly, we may determine the recovery paid out to more junior tranches.

Valuing EETCs within the Framework

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- With a slight modification to the general loan valuation formula presented earlier because of the priority given in recovery, the EETC valuation is almost exactly the same as a standard loan:

$$
\begin{aligned}
M \quad = \quad & \sum_{i=A,B,C} \sum_{k=1}^{N} \mathbb{E}\left[e^{-\int_0^{t_k} r_s ds} 1_{\{\tau > t_k\}} C_k^i\right] \\
& + \sum_{i=A,B,C} \sum_{k=1}^{N} \mathbb{E}[1_{\{t_{k-1} < \tau \leq t_k\}} e^{-\int_0^{\tau} r_s ds} (1 + (\tau - t_{k-1}) L_i) P_{k-1}^i] \\
& -(1-\pi) \sum_{k=1}^{N} \mathbb{E}\left[e^{-\int_0^{\tau} r_t dt} 1_{\{t_{k-1} < \tau \leq t_k\}} (K_k^{(ABC)}(\tau) - S(\tau))^+\right]
\end{aligned}
$$

where $\quad K_k^{(ABC)}(\tau) = TC + \sum_{i=A,B,C} (1 + (\tau - t_{k-1}) L_i) P_{k-1}^i$

Application

As an example, we implemented the analytical loan valuation model as applied to the 2007 EETC issued by Continental Airlines.

- Features
  - Collateralized by 12 Boeing 737-824 aircraft and 18 Boeing 737-924ER, delivered new.
  - 3 Tranches: A, B, C

Prospectus Date	4/10/2007
Asset Value	$1,554,200,000
Total Principal	$1,146,810,000
Average Aircraft Age at Issuance	NEW
Risk Free Rate A	5.29%
Prospectus Rate A	5.98%
Spread(bps)	70
Actual Maturity	15
LTV(A)	48.7%
Risk Free Rate B	5.26%
Prospectus Rate B	6.90%
Spread(bps)	164
Actual Maturity	15
LTV(B)	63.0%
Risk Free Rate C	5.14%
Prospectus Rate C	7.34%
Spread(bps)	220
Actual Maturity	7
LTV(C)	73.8%

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- Recovery Rate of $\pi = 0.19$ as estimated from airline industry historical default studies.

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Results

The calibration results yielded that $\rho = 0.72, \eta = 0.52$ for the 2007 EETC issued by Continental Airlines. Due to the risk adjustment, the equipment price depreciates faster under the risk-neutral measure $\mathbb{Q}$ than under the physical measure P. The risk adjusted residual value curve (RA-RVC) corresponding to the market price of risk parameter $\eta = 0.52$ is: